## On Finite $p$-Groups Whose Central Automorphisms Are All

 $n$th Class-Preserving
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## Abstract

Let $G$ be a finite non-abelian $p$-group, where $p$ is a prime. An automorphism $\alpha$ of $G$ is called an $n$th class-preserving if for each $x \in G$, there exists an element $g_{x} \in \gamma_{n}(G)$ such that $\alpha(x)=g_{x}^{-1} x g_{x}$. An automorphism $\alpha$ of $G$ is called a central automorphism if $x^{-1} \alpha(x) \in Z(G)$ for all $x \in G$. Let $\operatorname{Aut}_{c}^{n}(G)$ and Autcent $(G)$, respectively, denote the group of all $n$th class-preserving and central automorphisms of $G$. We give necessary and sufficient conditions for a finite $p$-group $G$ of class $n+1$ such that $\operatorname{Aut}_{c}^{n}(G)=\operatorname{Autcent}(G)$.

Keywords Central automorphism $\cdot n$th class-preserving automorphism
Mathematics Subject Classification 20D45 - 20D15

## 1 Introduction

Let $G$ be a finite $p$-group and let $Z(G)$ and $\Phi(G)$, respectively, denote the center and the Frattini subgroup of $G$. For a subgroup $H$ of $G$, let $x^{H}$ denote the subset $\left\{g^{-1} x g \mid g \in H\right\}$ of $G$. Notice that $x^{H}=x[x, H]$ and thus $\left|x^{H}\right|=|[x, H]|$ for all $x \in G$. We denote the subgroup $\left\langle x \in G \mid x^{p^{i}}=1\right\rangle$ of $G$ by $\Omega_{i}(G)$, where $i$ is a positive integer.

An automorphism $\alpha$ of $G$ is called central if $g^{-1} \alpha(g) \in Z(G)$ for all $g \in G$ and is called class-preserving if $\alpha(g) \in g^{G}$ for all $g \in G$. The set Autcent $(G)$ of all central automorphisms of $G$ and the set $\operatorname{Aut}_{c}(G)$ of all class-preserving automorphisms of $G$ are normal subgroups of $\operatorname{Aut}(G)$, where $\operatorname{Aut}(G)$ denotes the group of all automorphisms of $G$. Yadav [4, Theorem A] gave necessary and sufficient conditions on a finite $p$-group $G$ of class 2 such that $\operatorname{Aut}_{c}(G)=$ Autcent $(G)$. We call an automorphism $\alpha$ of $G$ an $n$th class-preserving if for all $g \in G, \alpha(g) \in g^{\gamma_{n}(G)}$, where $\gamma_{n}(G)$ denotes the

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