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SHORT COMMUNICATIONS

On Commuting Automorphisms of Finite p-Groups*

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1. INTRODUCTION

Let G be a finite p-group. A finite p-group G is called *extraspecial* if $G' = Z(G) \simeq C_p$. A finite p-group G is called *Frattinian* if $Z(G) \neq Z(M)$ for all maximal subgroups M of G. A Frattinian p-group G is called *strongly Frattinian* if $C_G(Z(\Phi(G))) = \Phi(G)$. A central automorphism of a group is an automorphism that commutes with all inner automorphisms. The central automorphism, fix the commutator subgroup G' of G elementwise and form a normal subgroup $\mathrm{Aut}_z(G)$ of $\mathrm{Aut}(G)$. The center $Z(\mathrm{Inn}(G))$ of the group of all inner automorphisms of G is always a subgroup of $\mathrm{Aut}_z(G)$. An automorphism α of G is called an IA-*automorphism* if $x^{-1}\alpha(x) \in G'$ for all $x \in G$. Let $\mathrm{IA}(G)$ denote the group of all IA-automorphisms of G. We call a group G *semicomplete* if the group of automorphisms which induce the identity on G/G' coincides with the group of inner automorphisms. If G is of order p^n and of nilpotency class c, then G is said to be of coclass $n - c$. The nilpotency class, second center and rank of G is denoted by $\mathrm{cl}(G)$, $Z_2(G)$ and $d(G)$ respectively. All other unexplained notations are standard.

Let G be a group. An automorphism α of G is called a *commuting automorphism* if $\alpha(x)x = x\alpha(x)$ for all $x \in G$. The set of all commuting automorphisms of G is denoted by $A(G)$. It is well-known that $A(G)$ does not necessarily form a subgroup of $\mathrm{Aut}(G)$, but it has a number of interesting properties (see [1]). In 1984, Herstein [2] proposed the following problem: If G is a simple non-Abelian group, then prove that $A(G) = 1$. In 1986, giving an answer to this problem, Laffey [3] proved that if G has no nontrivial Abelian normal subgroup, then $A(G) = 1$. Pettet [4], in a personal communication, observed that if $Z(G) = 1$ and $G' = G$, then $A(G) = 1$. Fouladi and Orfi [5] proved that if G is a finite AC group or a p-group of maximal class or metacyclic p-group, then G is an $A(G)$ group. A group G is called an *$A(G)$-group* if the set $A(G)$ is a subgroup of $\mathrm{Aut}(G)$. Vosooghpour and Malayeri [6] proved that if G is a finite p-group of of order p^n where $n \leq 4$, then G is an $A(G)$-group. They also proved that if p is a prime number and $n \geq 5$, then there exists a non-$A(G)$ group of order p^n.

In this short paper, we prove the following results: Let G be a finite non-Abelian p-group, p an odd prime such that $\mathrm{IA}(G) = \mathrm{Inn}(G)$ and $|G/G'| = p^2$. Then G is an $A(G)$-group. We also prove that if G is a finite non-Abelian p-group of coclass 3, where p is an odd prime and G is strongly Frattinian, then G is an $A(G)$-group.

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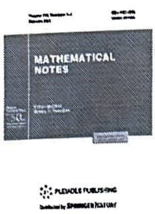

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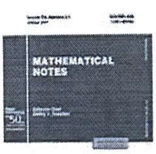

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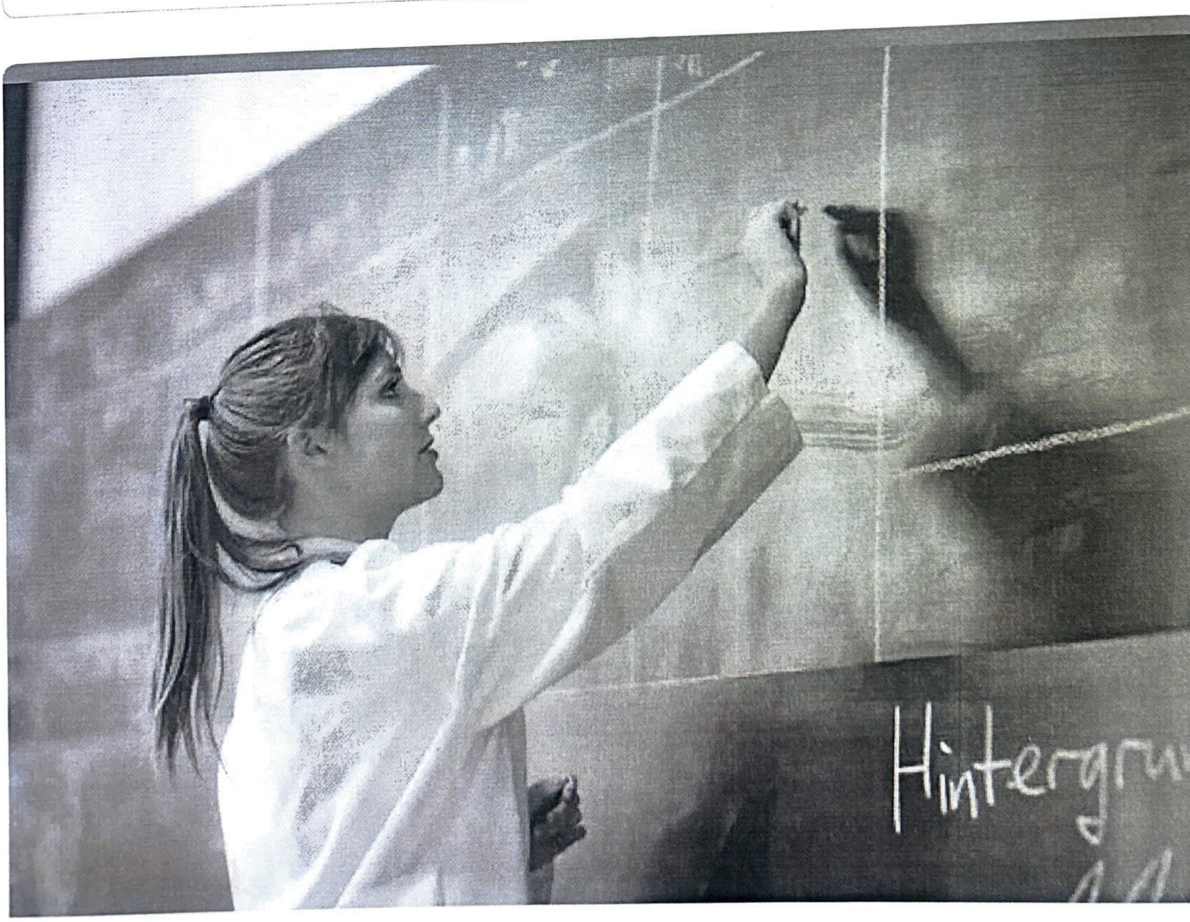

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