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SHORT COMMUNICATIONS

On Commuting Automorphisms of Finite p-Groups with a Metacyclic Quotient*

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Abstract—Let G be a finite non-Abelian p-group, where p is an odd prime, such that $G/Z(G)$ is metacyclic. We prove that all commuting automorphisms of G form a subgroup of $\mathrm{Aut}(G)$ if and only if G is of nilpotence class 2.

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Keywords: *commuting automorphism, metacyclic p-group.*

1. INTRODUCTION

Let G be a group. An automorphism α of G is called a *commuting automorphism* if $\alpha(x)x = x\alpha(x)$ for all $x \in G$. The set of all commuting automorphisms of G is denoted by $A(G)$. It is well known that $A(G)$ does not necessarily form a subgroup of $\mathrm{Aut}(G)$, but it has a number of interesting properties (see [1]). In 1984, Herstein [4] proposed the following problem: Is it true that if G is a simple non-Abelian group, then $A(G) = 1$? In 1986, giving an answer to this question, Laffey [6] proved that if G has no nontrivial Abelian normal subgroup, then $A(G) = 1$. Pettet (see [6]), in his personal communication, observed that if $Z(G) = 1$ and $G' = G$, then $A(G) = 1$. A group G is called an *$A(G)$-group* if the set $A(G)$ is a subgroup of $\mathrm{Aut}(G)$. The authors of [3, Theorem 4.2] proved that if G is a finite non-Abelian metacyclic p-group, then G is an $A(G)$-group. Observe that if G is metacyclic, then $G/Z(G)$ is also metacyclic. But the converse need not be true. This raises the obvious question: Is G an $A(G)$-group if $G/Z(G)$ is metacyclic? In this short paper, we prove the following result.

Theorem. *Let G be a finite non-Abelian p-group, where p is an odd prime, such that $G/Z(G)$ is metacyclic. Then G is an $A(G)$-group if and only if $cl(G) = 2$.*

2. PROOF OF THEOREM 1

Let G be a finite non-Abelian p-group such that $G/Z(G)$ is metacyclic. Then there is a normal subgroup $M/Z(G)$ of $G/Z(G)$ such that both $M/Z(G)$ and $(G/Z(G))/(M/Z(G)) = G/M$ are cyclic. Let $G/M = \langle aM \rangle$, and let $M/Z(G) = \langle bZ(G) \rangle$. Then $G = \langle a, b, Z(G) \rangle$. Since $G' \leq M = \langle b, Z(G) \rangle$, b commutes with every commutator, and hence $[a,b]^i = [a, b^i]$ for all $i \geq 1$. Let $|bZ(G)| = p^m$. Then $[a,b]^{p^m} = [a, b^{p^m}] = 1$. But for $0 < n < m$, we have $[a,b]^{p^n} = [a, b^{p^n}] \neq 1$. Hence $|[a,b]| = p^m$. We fix the above notation for the rest of the paper.

Lemma. *Let G be a finite non-Abelian p-group, where p is an odd prime, such that $G/Z(G)$ is metacyclic. Then*

(i) $G' = \langle [a,b] \rangle$;

(ii) *if $cl(G) = 2$, then G is an $A(G)$-group;*

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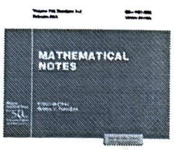

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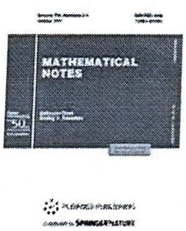

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